Gravity-wave detectors as probes of extra dimensions

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Abstract

If string theory is correct, then our observable Universe may be a 3-dimensional "brane" embedded in a higher-dimensional spacetime. This theoretical scenario should be tested via the state-of-the-art in gravitational experiments – the current and upcoming gravity-wave detectors. Indeed, the existence of extra dimensions leads to oscillations that leave a spectroscopic signature in the gravity-wave signal from black holes. The detectors that have been designed to confirm Einstein's prediction of gravity waves, can in principle also provide tests and constraints on string theory.

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Black holes are fascinating objects that are crucial to our theoretical understanding of gravity. They also provide a testing ground for gravitational theory, because they probe the strong-gravity regime. The deepest access to this regime is provided by gravity waves, which should be detected by the current and upcoming generation of experiments. These experiments open up a new opportunity to test candidate quantum gravity theories. Amongst these, string theory makes the radical prediction that spacetime has extra spatial dimensions – so that gravity propagates in higher dimensions and has extra polarizations. These polarizations would leave a "smoking gun" imprint in the gravity-wave signal from black hole events.

How do the extra spatial dimensions escape detection and observation at low energies? Recent developments in string theory provide a revolutionary new way to achieve this – the brane-world scenario, in which the observable Universe, including Standard Model fields, is confined to a 3-dimensional "brane", while gravity propagates in the full "bulk" spacetime [1, 2, 3]. The extra dimensions can be large (relative to the Planck length), and as a consequence, the true fundamental energy scale of gravity can be much lower than the 4-dimensional Planck scale, perhaps even down to the TeV-level. In that event, small black holes would be produced in the upcoming generation of particle colliders [4]. TeV-scale gravity would bring quantum gravity within reach of the laboratory. But if the fundamental scale is much higher (while still lower than the 4-dimensional Planck scale), then laboratory testing is pushed further into the future. By contrast, gravity-wave detectors provide access to black hole events at energies high enough to carry any detectable signatures of extra-dimensional gravity.

The brane-world scenario encompasses a very wide variety, including string theory solutions and phenomenological models. In order to investigate perturbations of black holes that have formed via gravitational collapse, one needs a model which is simple enough to have an exact background solution, but rich enough to include key aspects of astrophysical black holes and of string theory. Perhaps the best candidate is the black string solution [5], in which the 4-dimensional Schwarzschild metric is embedded in a 5-dimensional Randall-Sundrum type model [2]. One important feature of the Randall-Sundrum scenario is that the self-gravity of the brane is included, which leads to a curvature ("warping") of the extra dimension.

In the original Randall-Sundrum model, two Minkowski branes, with equal and opposite tensions (vacuum energies) enclose a patch of 5-dimensional anti de Sitter spacetime, with curvature scale ℓ. The bulk metric satisfies the 5-dimensional Einstein equations, $G_{ab} = 6g_{ab}/\ell^2$, and the boundary conditions at the branes are the Israel junction conditions. There is a mirror symmetry at each brane (a feature of some string theory solutions). The black string metric satisfies the same equations in the bulk, and is given by

$$ds^2 \;=\; \mathrm{e}^{-2|y|/\ell}\left[-(1-2GM/r)dt^2+\frac{dr^2}{1-2GM/r}+r^2\,d\Omega^2\right]+dy^2\,. \tag{1}$$

The induced metric on the "visible" brane at $y=0$ is the Schwarzschild metric. If the horizon

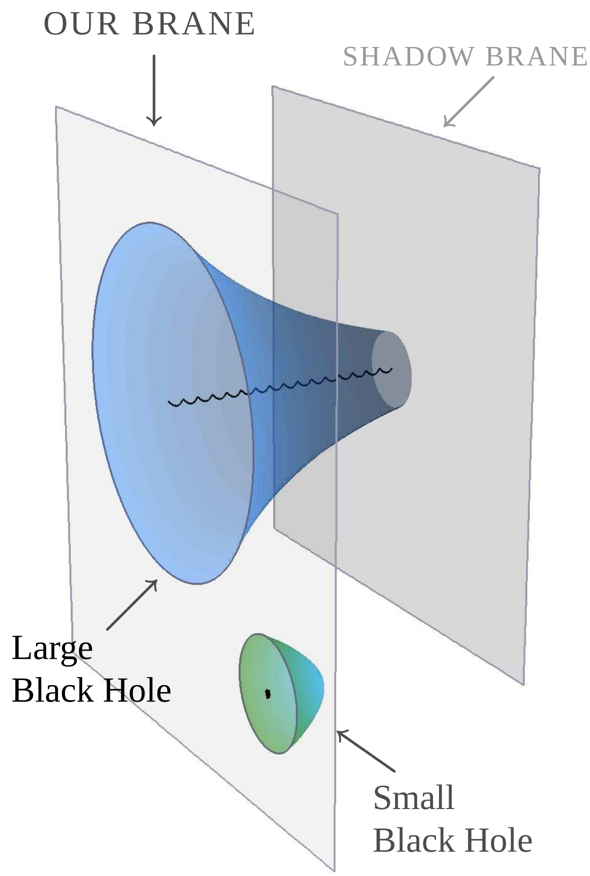

Figure 1: Schematic of brane-world black holes. (The brane separation is greatly exaggerated.)

radius on the brane is much greater than the size of the extra dimension, $2GM \gg d$, then we can think of the extension of the horizon into the 5th dimension as being cut off by the "shadow" brane at $y = d$ [6]. By contrast, small black holes, $2GM \ll d$, do not "see" the brane and behave like 5-dimensional black holes, localized on the visible brane. No exact solution is known for a black hole localized on the brane, but the nature of small localized black holes has been investigated numerically [7]. This set-up is illustrated in Fig. 1.

The shadow brane simultaneously provides an infrared cut-off to shut down the Gregory-Laflamme long-wavelength instability of the black string. The condition for stability is [8]

$$GM \gtrsim 0.1\,\ell\,\mathrm{e}^{d/\ell}\,. \tag{2}$$

In fact small black holes ($GM < 0.1\ell\mathrm{e}^{d/\ell}$) localized on the brane can be seen as products of the instability, since there is evidence for a black string to black hole transition as the outcome of the instability [9]. The parameters are also constrained by solar system observations and by laboratory tests of Newton's law, via a remarkable interlinking of different gravitational

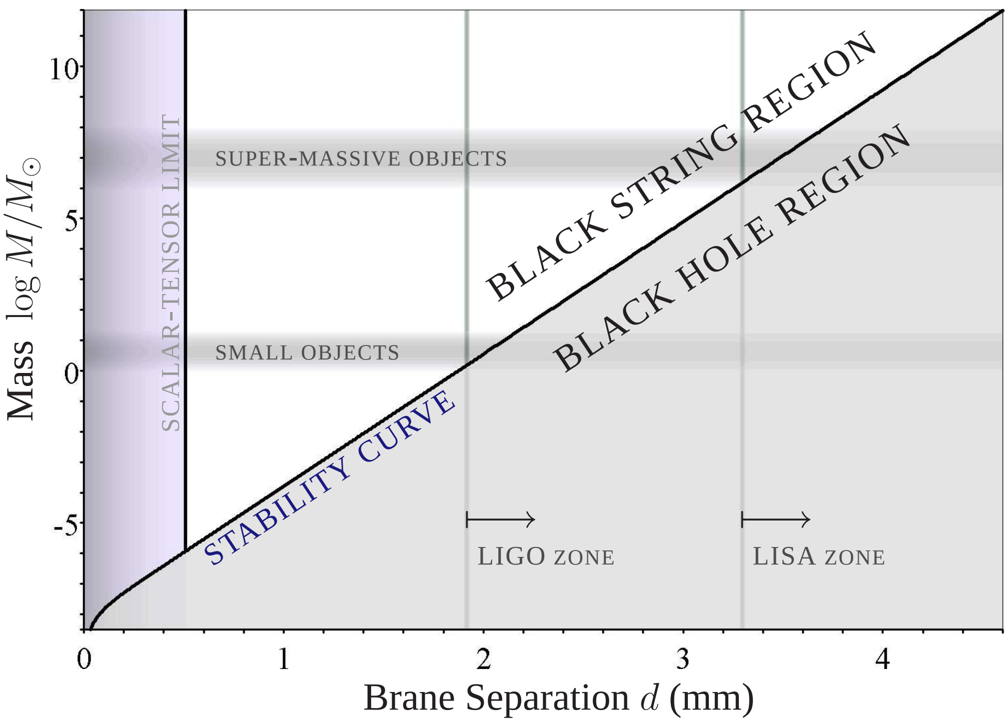

Figure 2: The parameter space of the black string, for $\ell = 0.1$mm. The stability curve is approximated by Eq. (2). The LIGO and LISA zones follow from Eq. (4).

properties. The shadow brane must be far enough away that its gravitational influence on the visible brane is within observational limits. The brane separation is a massless scalar degree of freedom (the "radion") on the visible brane, so that the effective theory on the visible brane is of Brans-Dicke type [10], with $\omega_{\rm bd} = 3(e^{2d/\ell} - 1)/2$. Solar system observations impose the lower limit [11, 12] $\omega_{\rm bd} \gtrsim 4 \times 10^4$, so that $d/\ell \gtrsim 5$. The allowed region in parameter space is shown in Fig. 2. Table-top tests of Newton's law impose the constraint [13] $\ell \lesssim 0.1\,$mm, and we use this upper limit in Fig. 2.

We also show in Fig. 2 the detectable minimum brane separation, corresponding to a maximum characteristic frequency, for the LIGO and LISA detectors. The extra-dimensional polarizations of the graviton are realized on the visible brane as effectively massive Kaluza-Klein modes, which produce late-time oscillations in the gravity-wave signal, as described below. Because of the boundary conditions at the branes, the Kaluza-Klein masses form a discrete tower [2]

$$m_n = (z_n/\ell)e^{-d/\ell} \text{ where } J_1(z_n) \approx 0\,,\ n = 1, 2, 3, \cdots \tag{3}$$

The corresponding tower of frequencies for the oscillating massive modes is [14]

$$f_n = z_n e^{26.9 - d/\ell}(0.1\,\text{mm}/\ell)\ \text{Hz}. \tag{4}$$

The z_n's have a spacing of order one. It is striking that, unlike 4-dimensional black hole quasinormal modes, these frequencies are *independent* of the mass M. For $\ell = 0.1\,\mathrm{mm}$, the lowest frequency f_1 is within the LIGO upper limit $f_{\max} \sim 10^4\,\mathrm{Hz}$ for $d/\ell \sim 20$. The LISA upper limit gives $d/\ell \sim 30$. Typical optimal frequencies are

$$\begin{aligned} &\text{LIGO:} \quad f_1 \sim 100\,\mathrm{Hz} \text{ which implies } M > 100 M_\odot\,, \\ &\text{LISA:} \quad f_1 \sim 0.01\,\mathrm{Hz} \text{ which implies } M > 10^6 M_\odot\,. \end{aligned}$$

Figure 2 also shows that the detectors can in principle see a black string instability event for small or supermassive objects, which would be one of the best sources of massive modes.

Current detectors fall into two categories: those that are very nearly up-and-running, such as LIGO, TAMA and GEO, detect high frequencies around 1kHz, while proposed space detectors such as LISA are designed for low frequencies around the mHz range. All are designed to cover two or three orders of magnitude around these values. Will they be able to detect massive modes of higher-dimensional gravitons, if such modes exist? The smallest d/ℓ these detectors can measure is illustrated by the vertical bands in Fig. 2 for high- and low-frequency detectors. But these detectors can do rather better than just the f_1 mode: the tower of frequencies (4) is dense enough to allow about ten discrete mass modes in each decade. Thus all brane separations to the right of the vertical bands will have a range of around 30 massive mode frequencies detectable by present technology.

The gravity-wave output on the brane from black string events is in general a combination of the massless zero-mode, $m = 0$, which reproduces the standard 4-dimensional signal (a damped single-frequency quasinormal ringing followed by a power-law tail [15]), and the massive Kaluza-Klein modes, with much less damping and late-time oscillations [14]. The details follow from solving the wave equation for perturbations of the metric (1), which reduces to five coupled linear wave equations on the brane, corresponding to the 5 polarizations of the 5-dimensional graviton (and generalizing the 2-dimensional system of the Regge-Wheeler and Zerilli equations [15]).

The results of simulations [14] are shown in Fig. 3, for the $L = 2$ axial modes (in a generalized Regge-Wheeler gauge), in the case of initial data that weakly excites the lowest massive modes – for example, an encounter between a heavy black string and a small localized black hole or other body on the brane. In General Relativity the late-time signal is featureless – a simple power-law tail. For the black string however, an observer would see a high-frequency signal that persists at late times. The Fourier transform of this late-time signal shows *a spectroscopic series of spikes, located at the characteristic frequencies of the massive modes.*

Initial data that strongly excites the massive modes corresponds to events which "mainly take place in the bulk", such as the merger of two black strings. In this case there is very similar late-time behaviour, but the early-time waveform has no General Relativity-like ringdown. In all cases, the key common feature is that *the signal is made of discrete frequencies which are independent of the mass of the black string.*

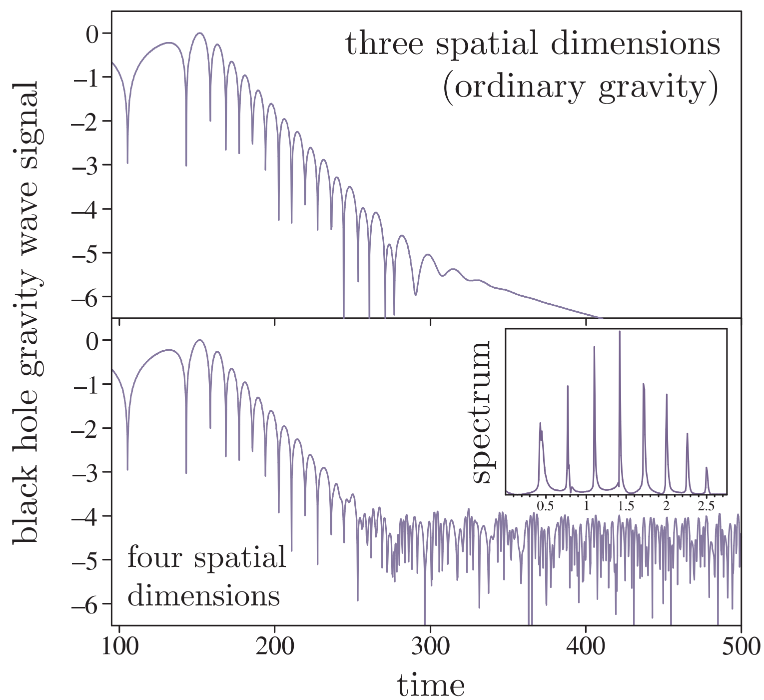

Figure 3: Gravity-wave signal from a black hole in General Relativity (upper panel) and a brane-world black string (lower panel). In the black string case, the early-time waveform is dominated by the zero-mode (the General Relativity ringdown), but at late times the massive modes take over and remove the General Relativity power-law tail. The Fourier transform of the late-time signal is shown in the inset.

Can the massive signal be resolved by gravity-wave detectors? This entails both the frequency of the massive modes, discussed above, and their relative amplitude. The relative amplitude of the massive to massless signal depends strongly on whether the initial data has primary support on the brane or in the bulk. For the brane-based perturbation in Fig. 3, the detection of the tail would require a signal-to-noise ratio $\gtrsim 10^4$, just beyond the projected capabilities of LISA [12]. The relative massive amplitude tends to increase with decreasing d/ℓ. However, small d/ℓ implies large f_n, so the best prospect of seeing this type of signal lies with high frequency detectors. Such constraints do not apply to bulk-based initial data, like black string mergers. Since the zero-mode energy is small in these situations, we expect the strength of the massive mode oscillations to be comparable to the quasinormal ringing amplitude in the analogous 4-dimensional case.

At late times and large distances on the brane from the black string, the massive modes decay very slowly, as $t^{-5/6}$ – unlike the exponential decay of the zero mode. Other than direct observation of an event, what consequences could this gentle decay have? A black string which is being continually excited will steadily emit gravity waves, typically with some excitation of the massive modes. For example, for two black strings rotating around each other in the latter stages of merger, massive modes would be continually produced. An observer would see a signal composed of a mixture of a General Relativity inspiral waveform, interlaced with a massive signal. After merger the massive modes would persist for much longer than the massless mode – for a massive mode, the signal strength drops by a factor of 10 after ~ 16 times the lifetime of the massless mode. Typically there would be many events like this taking place in a galaxy over time. As the signal from one merger never properly decays away, massive modes of the higher-dimensional graviton would accumulate over time, reaching an equilibrium strength proportional to the number of mergers. This would lead to *an integrated massive mode contribution to the stochastic gravity-wave background.* The gravity-wave background would in principle give access to the massive mode frequencies.

Furthermore, since the massive modes travel below light-speed, there will be potentially observable *time-delays* in their arrival, which could be of order seconds or longer for distant sources.

The discrete nature of the late-time Fourier transform of the black string waveform is the most important observable feature. Its detection would provide clear evidence of extra dimensions, and could further give the direct spectroscopic measurement of the Kaluza-Klein masses m_n. This in turn provides information about the size and shape of the extra dimension – gravity-wave detectors can in principle explore the geography of large extra dimensions.

These results are based on a specific simple model of brane-world black holes (the only exact solution with reasonable astrophysical properties that we know of). But we expect that qualitatively similar features will arise for other models with large extra dimensions, since they all have a discrete tower of massive Kaluza-Klein modes. Could it be possible that we will be able to observe or exclude predictions of string theory using current technology? We might not have to wait long to find out.

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